A Fixed-Parameter Algorithm for Max Edge Domination

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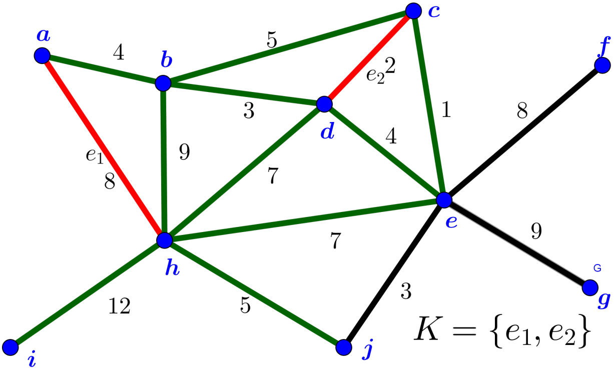
Max Edge Domination

Edge domination : An edge e dominates itself and its adjacent edges.
$D(e)$: Edge set dominated by an edge e
$D(K)$: Edge set dominated by the edge set K



Max Edge Domination

Max Edge Domination(MaxED)

- Input : graph $G = (V, E)$, edge weight w, integer k
- Output : a subset $K \subseteq E$ with cardinality at most k such that total weight of edges dominated by K is maximized

Formulation

$$\max_{K} \sum_{e \in D(K)} w_e \text{ subject to } K \subseteq E, |K| \leq k$$

- NP-hard (From NP-hardness of minimum edge dominatings set problem[Yannakakis and Gavril])
- APX-hard
- $\max\{(1 - \frac{1}{e}), \frac{k}{s}\}$-approximability ($s$ is the size of maximal matching)

Fixed parameter Tractability

Definition (Fixed parameter Tractability)

Given an instance of size n and a parameter γ, if it can be solved in $f(\gamma) \cdot n^{\mathrm{O}(1)}$-time, then it is Fixed Parameter Tractable(FPT).

Parameter		k (Solution size)		ω (treewidth)
graph		general	apex-minor-free	general
VC	Min	FPT	Subexponential FPT ($2^{O(\sqrt{k})} \cdot n^{O(1)}$)	FPT
	Max	$W[1]$-hard		
DS	Min	$W[2]$-hard		
	Max	$W[2]$-hard		
ED	Min	FPT		
	Max	$W[1]$-hard	$2^{O(\sqrt{k})} \cdot n^{O(1)}$	$O(3^{\omega} \cdot k \cdot n \cdot (k + \omega^2))$

VC:Vertex Cover, DS:Dominating Set, ED:Edge Dominating set

Result & Future Work

Technique

- Dynamic Programming algorithm based on nice tree decomposition
- Bidimensionality

Result

- There is an $O(3^{\omega} \cdot k \cdot n \cdot (k + \omega^2))$-time algorithm for Max Edge Domination on general graph.
- There is an $2^{O(\sqrt{k})} \cdot n^{O(1)}$-time algorithm for Max Edge Domination on apex-minor-free graph.

Future Work

- Approximation approach for Max Edge Domination